Data-Flow Analysis

Proving Little Theorems

Data-Flow Equations

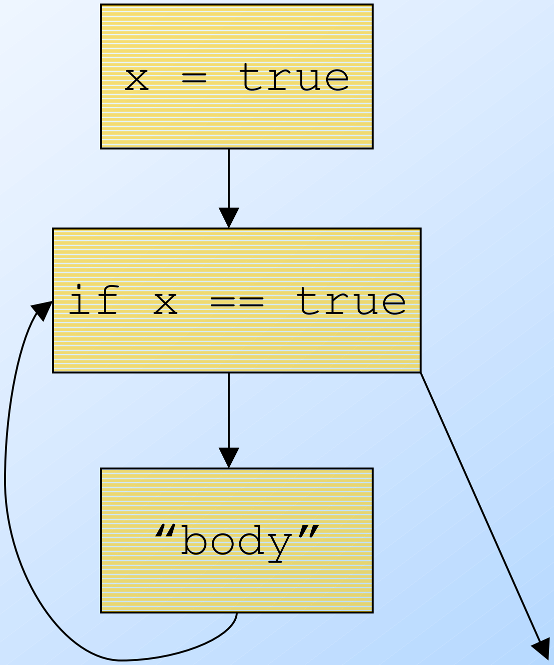
Major Examples

An Obvious Theorem

```
boolean x = true;
while (x) {
   . . . // no change to x
}
```

- Doesn't terminate.
- Proof: only assignment to x is at top, so x is always true.

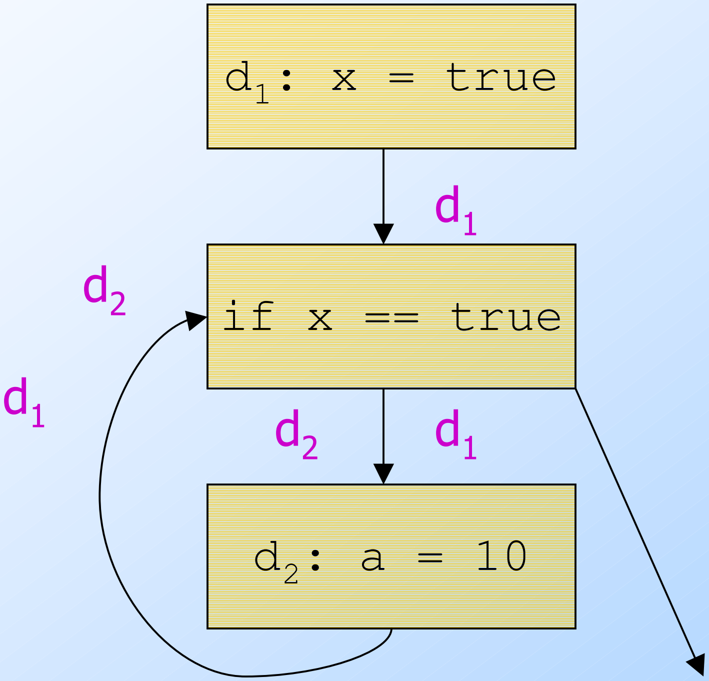
As a Flow Graph



Formulation: Reaching Definitions

- Each place some variable x is assigned is a *definition*.
- Ask: for this use of x, where could x last have been defined.
- In our example: only at `x=true`.

Example: Reaching Definitions



Clincher

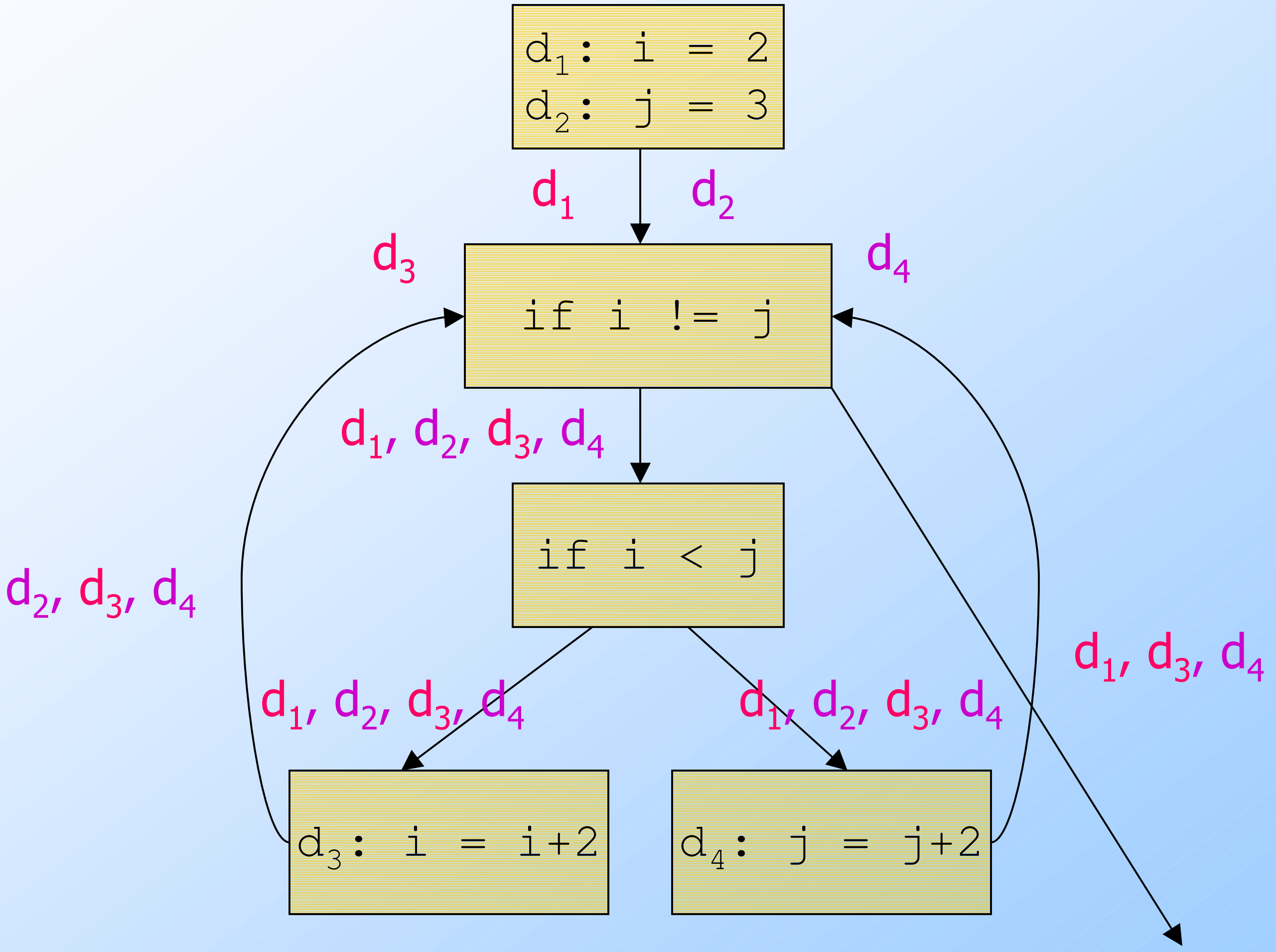
- Since at `x == true`, d_1 is the only definition of x that reaches, it must be that x is true at that point.
- The conditional is not really a conditional and can be replaced by a branch.

Not Always That Easy

```
int i = 2; int j = 3;
while (i != j) {
    if (i < j) i += 2;
    else j += 2;
}
```

- We'll develop techniques for this problem, but later …

The Flow Graph



d_1: i = 2
d_2: j = 3

d_1 d_2

d_3 d_4

if i != j

d_1, d_2, d_3, d_4

if i < j

d_2, d_3, d_4

d_1, d_2, d_3, d_4 d_1, d_2, d_3, d_4

d_1, d_3, d_4

d_3: i = i+2

d_4: j = j+2

DFA Is Sufficient Only

- In this example, i can be defined in two places, and `j` in two places.
- No obvious way to discover that `i!=j` is always true.
- But OK, because reaching definitions is sufficient to catch most opportunities for *constant folding* (replacement of a variable by its only possible value).

Be Conservative!

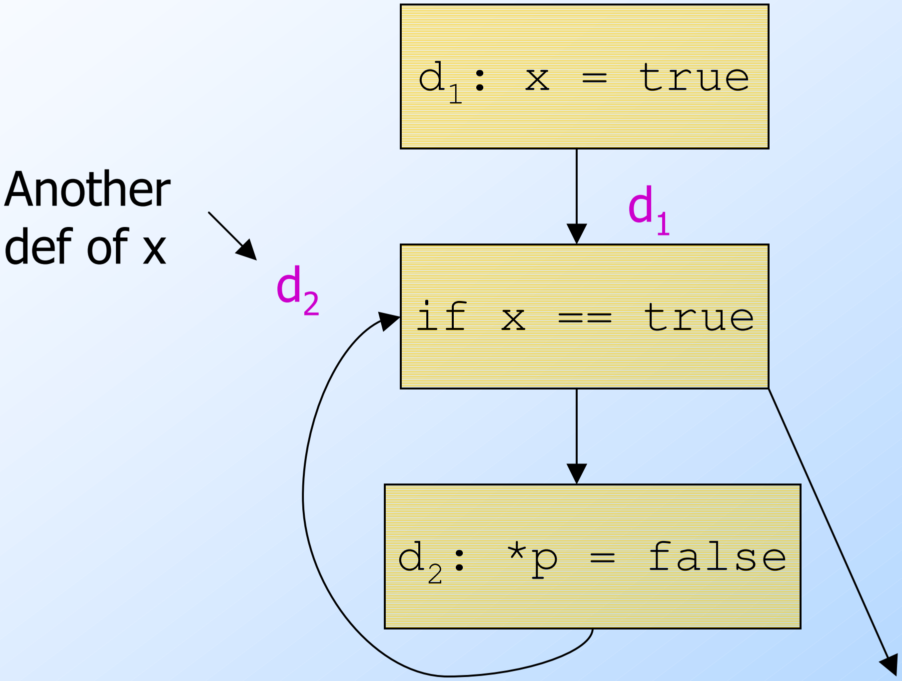
- (Code optimization only)
- It's OK to discover a subset of the opportunities to make some code-improving transformation.
- It's not OK to think you have an opportunity that you don't really have.

Example: Be Conservative

```
boolean x = true;
while (x) {
   . . . *p = false; . . .
}
```

- Is it possible that p points to x?

As a Flow Graph



`d`$_1$`: x = true`

d_1

Another def of x

d_2

`if x == true`

`d`$_2$`: *p = false`

Possible Resolution

- Just as data-flow analysis of "reaching definitions" can tell what definitions of x might reach a point, another DFA can eliminate cases where p definitely does not point to x.
- Example: the only definition of p is `p = &y` and there is no possibility that y is an alias of x.

Reaching Definitions Formalized

- A definition d of a variable x is said to *reach* a point p in a flow graph if:
  1. Every path from the entry of the flow graph to p has d on the path, and
  2. After the last occurrence of d there is no possibility that x is redefined.

Data-Flow Equations --- (1)

- A basic block can *generate* a definition.
- A basic block can either
  1. *Kill* a definition of x if it surely redefines x.
  2. Transmit a definition if it may not redefine the same variable(s) as that definition.

Data-Flow Equations --- (2)

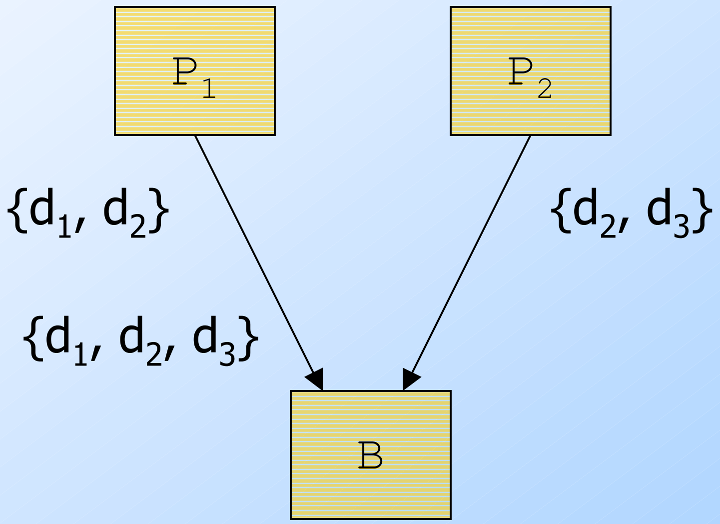
- Variables:
  1. IN(B) = set of definitions reaching the beginning of block B.
  2. OUT(B) = set of definitions reaching the end of B.

Data-Flow Equations --- (3)

- Two kinds of equations:
  1. Confluence equations : IN(B) in terms of outs of predecessors of B.
  2. Transfer equations : OUT(B) in terms of of IN(B) and what goes on in block B.

Confluence Equations

$$IN(B) = \cup_{\text{predecessors P of B}} OUT(P)$$



Transfer Equations

- *Generate* a definition in the block if its variable is not definitely rewritten later in the basic block.
- *Kill* a definition if its variable is definitely rewritten in the block.
- An internal definition may be both killed and generated.

Example: Gen and Kill

IN = {$d_2(x)$, $d_3(y)$, $d_3(z)$, $d_5(y)$, $d_6(y)$, $d_7(z)$}

Kill includes {$d_1(x)$, $d_2(x)$, $d_3(y)$, $d_5(y)$, $d_6(y)$,…}

d_1: y = 3
d_2: x = y+z
d_3: *p = 10
d_4: y = 5

Gen = {$d_2(x)$, $d_3(x)$, $d_3(z)$,…, $d_4(y)$}

OUT = {$d_2(x)$, $d_3(x)$, $d_3(z)$,…, $d_4(y)$, $d_7(z)$}

Transfer Function for a Block

- For any block B:

$$OUT(B) = (IN(B) - Kill(B)) \cup Gen(B)$$

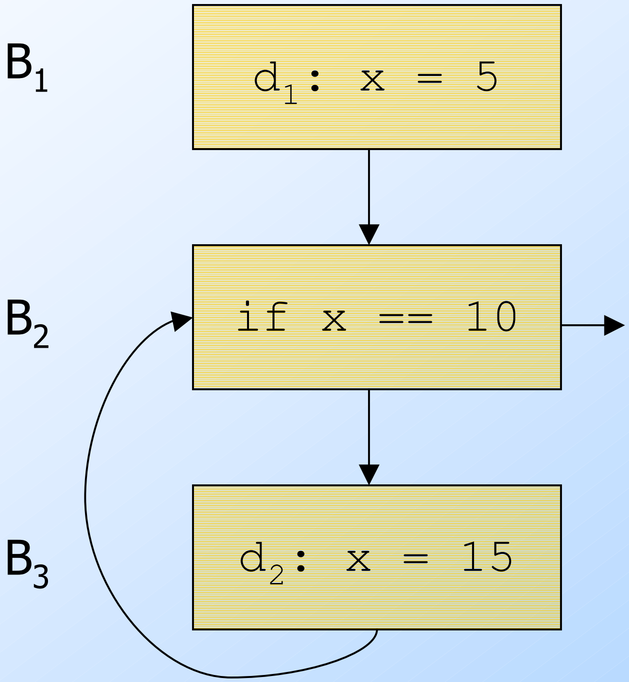
Iterative Solution to Equations

- For an n-block flow graph, there are 2n equations in 2n unknowns.
- Alas, the solution is not unique.
  - Standard theory assumes a field of constants; sets are not a field.
- Use iterative solution to get the least fixedpoint.
  - Identifies any def that might reach a point.

Iterative Solution --- (2)

```
IN(entry) = $\emptyset$;
for each block B do OUT(B)= $\emptyset$;
while (changes occur) do
  for each block B do {
    $IN(B) = \cup_{\text{predecessors } P \text{ of } B} OUT(P)$;
    $OUT(B) = (IN(B) - Kill(B)) \cup Gen(B)$;
  }
```

Example: Reaching Definitions



$\mathrm{IN}(B_1) = \{\}$

$\mathrm{OUT}(B_1) = \{ d_1 \}$

$\mathrm{IN}(B_2) = \{ d_1, d_2 \}$

$\mathrm{OUT}(B_2) = \{ d_1, d_2 \}$

$\mathrm{IN}(B_3) = \{ d_1, d_2 \}$

$\mathrm{OUT}(B_3) = \{ d_2 \}$

Aside: Notice the Conservatism

- Not only the most conservative assumption about when a def is killed or gen'd.
- Also the conservative assumption that any path in the flow graph can actually be taken.
- Fine, as long as the optimization is triggered by limitations on the set of RD's, not by the assumption that a def does not reach.

Another Data-Flow Problem: Available Expressions

- An expression x+y is *available* at a point if no matter what path has been taken to that point from the entry, x+y has been evaluated, and neither x nor y have even possibly been redefined.
- Useful for global common-subexpression elimination.

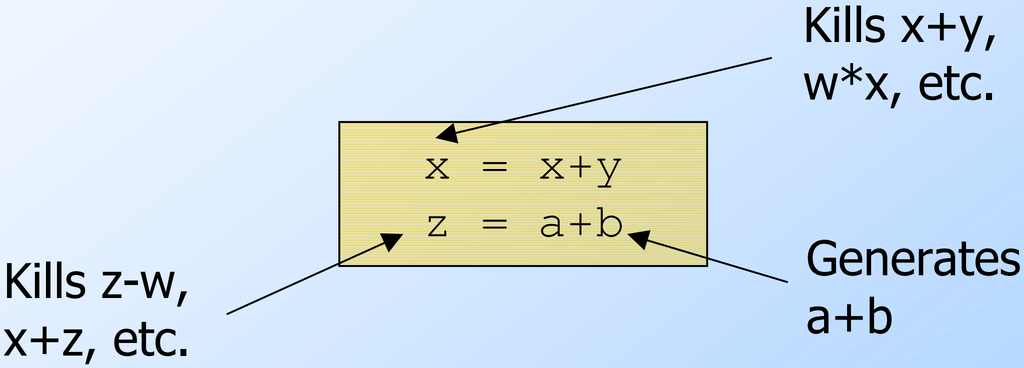
Equations for AE

- The equations for AE are essentially the same as for RD, with one exception.
- Confluence of paths involves intersection of sets of expressions rather than union of sets of definitions.

Gen(B) and Kill(B)

- An expression x+y is *generated* if it is computed in B, and afterwards there is no possibility that either x or y is redefined.
- An expression x+y is *killed* if it is not generated in B and either x or y is possibly redefined.

Example



```
x = x+y
z = a+b
```

Kills x+y, w*x, etc.

Kills z-w, x+z, etc.

Generates a+b

Transfer Equations

- Transfer is the same idea:

$$OUT(B) = (IN(B) - Kill(B)) \cup Gen(B)$$

Confluence Equations

- Confluence involves intersection, because an expression is available coming into a block if and only if it is available coming out of each predecessor.

$$IN(B) = \cap_{\text{predecessors } P \text{ of } B} OUT(P)$$

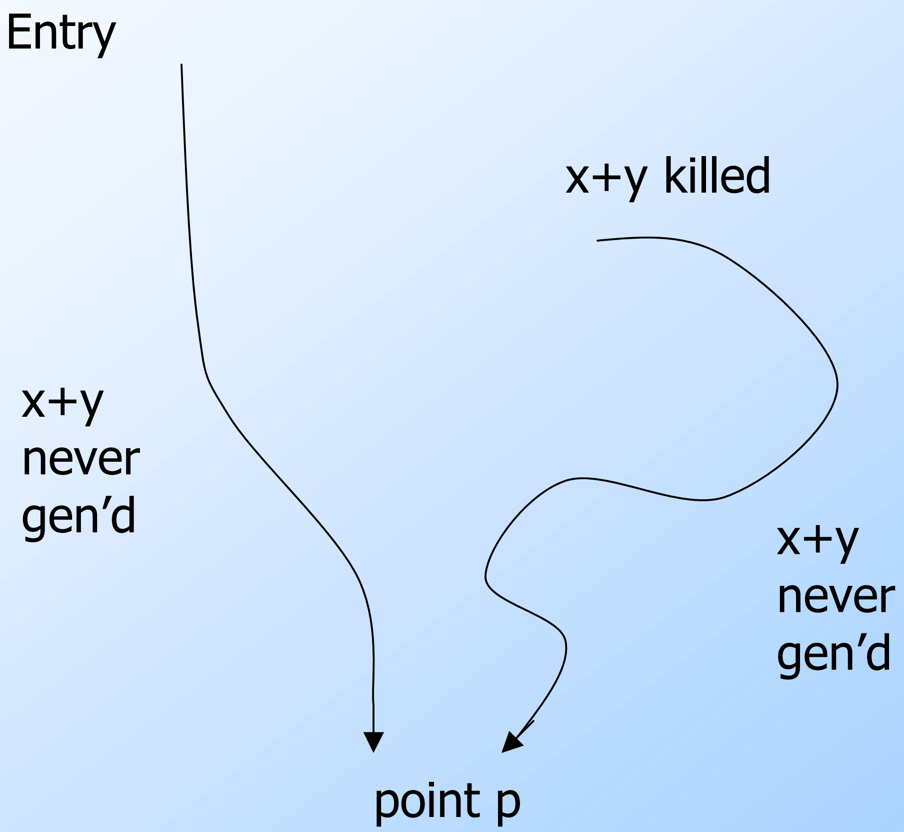
Iterative Solution

```
IN(entry) = ∅;
for each block B do OUT(B)= ALL;
while (changes occur) do
  for each block B do {
    $IN(B) = \cap_{\text{predecessors P of B}} OUT(P);$
    $OUT(B) = (IN(B) - Kill(B)) \cup Gen(B);$
  }
```

Why It Works

- An expression x+y is unavailable at point p iff there is a path from the entry to p that either:
  1. Never evaluates x+y, or
  2. Kills x+y after its last evaluation.
- IN(entry) = ∅ takes care of (1).
- OUT(B) = ALL, plus intersection during iteration handles (2).

Example



Entry

x+y killed

x+y
never
gen’d

x+y
never
gen’d

point p

Subtle Point

- It is conservative to assume an expression isn't available, even if it is.
- But we don't have to be "insanely conservative."
  - If after considering all paths, and assuming x+y killed by any possibility of redefinition, we still can't find a path explaining its unavailability, then x+y is available.

Live Variable Analysis

- Variable x is *live* at a point p if on some path from p, x is used before it is redefined.
- Useful in code generation: if x is not live on exit from a block, there is no need to copy x from a register to memory.

Equations for Live Variables

- LV is essentially a "backwards" version of RD.
- In place of Gen(B): Use(B) = set of variables x possibly used in B prior to any certain definition of x.
- In place of Kill(B): Def(B) = set of variables x certainly defined before any possible use of x.

Transfer Equations

- Transfer equations give IN's in terms of OUT's:

$$IN(B) = (OUT(B) - Def(B)) \cup Use(B)$$

Confluence Equations

- Confluence involves union over successors, so a variable is in OUT(B) if it is live on entry to any of B’s successors.

$$\mathrm{OUT}(B) = \cup_{\text{successors } S \text{ of } B} \mathrm{IN}(S)$$

Iterative Solution

```
OUT(exit) = $\emptyset$;
for each block B do IN(B)= $\emptyset$;
while (changes occur) do
   for each block B do {
      $OUT(B) = \cup_{\text{successors } S \text{ of } B} IN(S)$;
      $IN(B) = (OUT(B) - Def(B)) \cup Use(B)$;
   }
```